Topological aspects of Cardy-Rabinovici model

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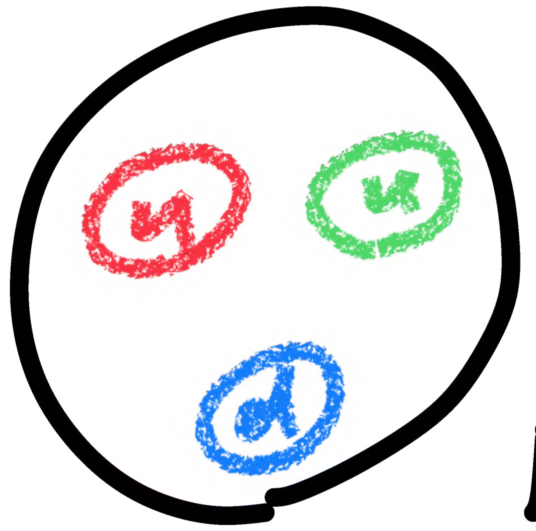
Based on 2009.10183 with Masazumi Honda (YITP)

Motivation Yang-Mills theory & θ-angles

Quark confinement

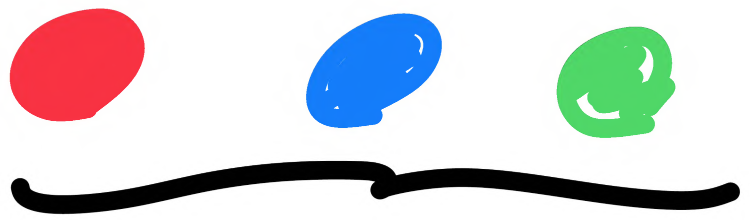
Strong interaction = SU(3) non-Abelian gauge theory



Proton

confined

Quark (fermion) is in its fund. rep.



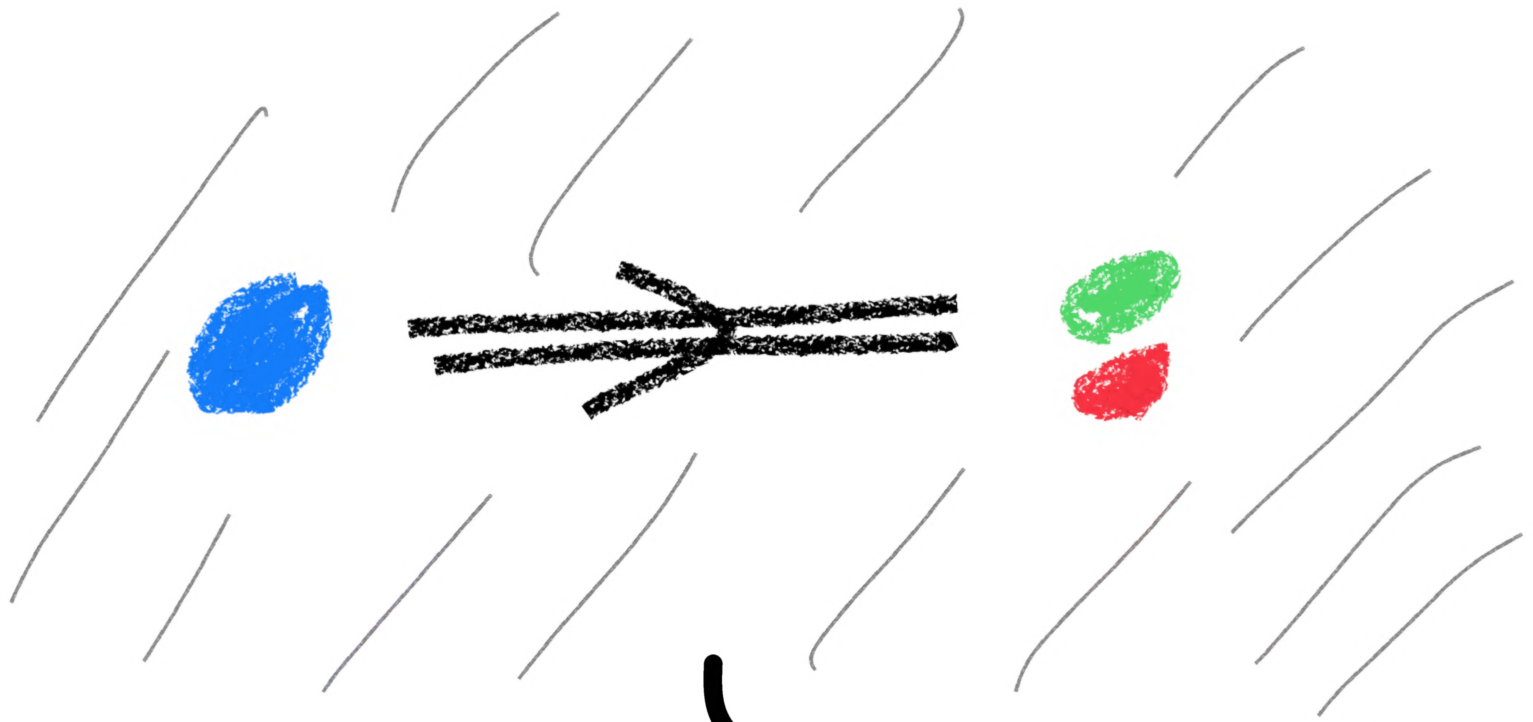
3-color.

Why such colored particles does not produce asymptotic states?

Monopole condensation.

Nambu, Mandelstam, 't Hooft ('70s)

Confinement is caused by dual superconductivity.



magnetic condensation.

Bunch of monopoles condense into the vacuum.

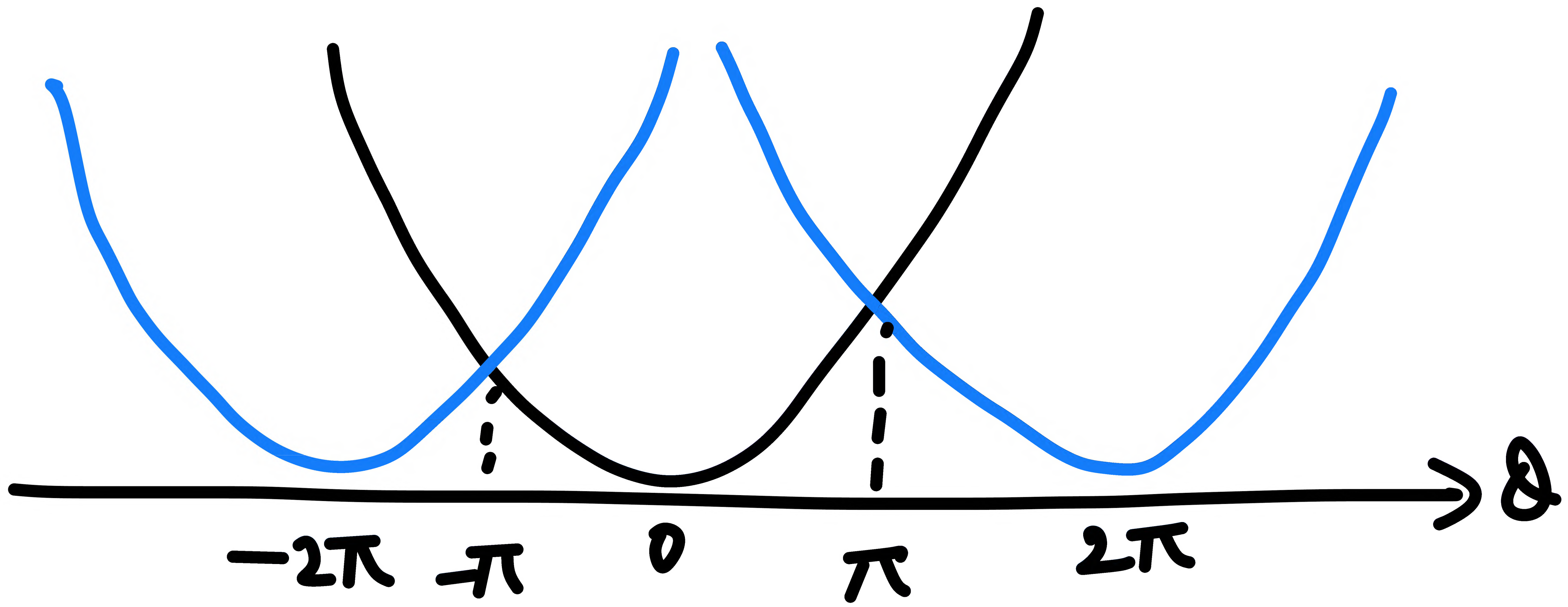
Such vacuum is also interesting from topological aspects.

Nontrivial topology

$SU(N)$ Yang-Mills theory : Theory of gluons

$$\mathcal{L} = \frac{1}{g^2} \mathrm{tr}(F_{\mu\nu} F_{\mu\nu}) + i\theta \underbrace{\frac{1}{8\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr}(F_{\mu\nu} F_{\rho\sigma})}_{\substack{\text{instanton \#} \\ = \text{winding \#} \in \mathbb{Z}}}$$

Vacuum energy has a nontrivial θ dependence :



(Witten '80, '98, Di Vecchia, Veneziano '80)

Large-N limit & Spontaneous CP breaking

Large-N : $N \to \infty$ with $\lambda = g^2 N$ fixed.

$$e^{-\beta \cdot \mathrm{Vol} \cdot E(\theta)} = \int \mathcal{D}a \, e^{-N\left(\frac{1}{2\lambda}\int F\wedge * F + i\frac{\theta/N}{8\pi^2}\int F\wedge F\right)}$$

d.o.f.

$$\Rightarrow E(\theta) = N^2 f\left(\frac{\theta}{N}\right)$$

$$= N^2 f(0) + \frac{\chi_{top}}{2}\theta^2 + O\left(\frac{1}{N^2}\right)$$

To achieve $\theta \sim \theta + 2\pi$,

$$E(\theta) = \min_n \left(\frac{\chi_{top}}{2}(\theta - 2\pi n)^2\right).$$

$\Rightarrow$ $n = 0$ & 1 have the same energy at $\theta = \pi$. $\cancel{CP}$

Anomaly matching

Recently, a new anomaly has been found for YM :

B : $\mathbb{Z}_N$ 2-form gauge field (= 't Hooft flux)

$$Z_{\theta=\pi}[B] \xrightarrow{CP} e^{-i\frac{N}{4\pi}\int B\wedge B} Z_{\theta=\pi}[B]$$

(Gaiotto, Kapustin, Komargodski, Seiberg '17, Tanizaki, Kikuchi '17, ...)

$\rightarrow$ ~~CP~~ is indeed natural to satisfy the anomaly matching.

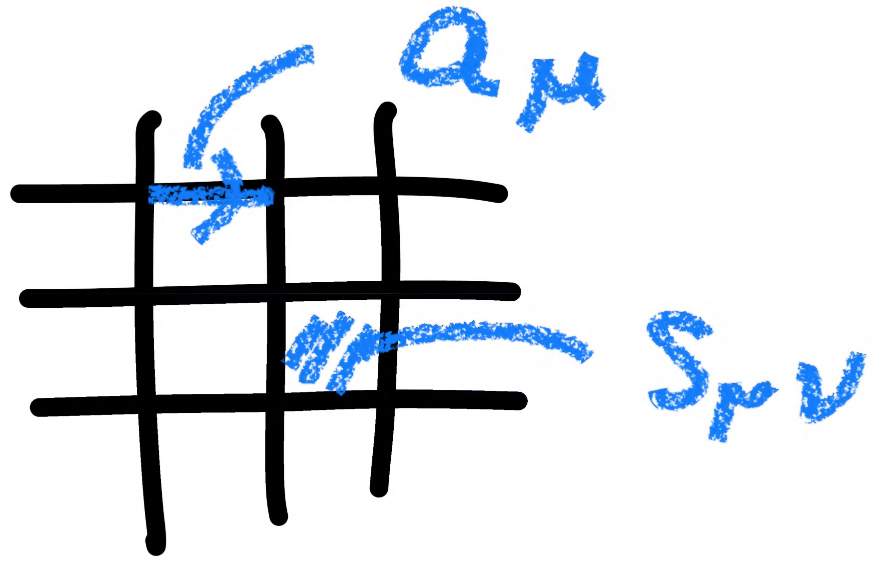
Q.) what are other candidates to achieve anomaly matching?

Model Cardy-Rabinovici model

Lattice U(1) gauge theory w/ confinement-deconfinement

(Banks, Myerson, Kogut '77
Savit '77 ...)

$$\begin{cases} a_\mu : \mathbb{R}\text{-valued link variable} \\ S_{\mu\nu} : \mathbb{Z}\text{-valued plaquette variable} \end{cases}$$



$\Rightarrow \quad f = da - 2\pi S$: field strength

$$\mathcal{L} = \frac{1}{g^2}\int f \wedge * f + iN \int \underline{n_\mu}\, a_\mu$$

world-line of electric charges

* Theory inherits magnetic particles

$$m = \frac{1}{2\pi} * df = * dS .$$

Cardy-Rabinovici ('82) : θ-angle via Witten effect "$n_\mu \Rightarrow n_\mu + \frac{\theta}{2\pi} m_\mu$".

Cardy - Rabinovici model

In a (formal) continuum description,

Wilson line ; 't Hooft line

$$Z = \int \mathcal{D}a \, \exp \int \left(-\frac{1}{2g^2}|f|^2 + i \frac{N\theta}{8\pi^2} \int f \wedge f \right) \times \sum_{\substack{\text{world-lines} \\ C_e, C_m}} W^N(C_e) \cdot H(C_m)$$

Witten effect

$$(Nn, m) \xrightarrow{\theta} \left(N\left(n + \frac{\theta}{2\pi} m\right), \; m \right).$$

Vacuum energy of CR model

If a particle of (n,m) condenses,

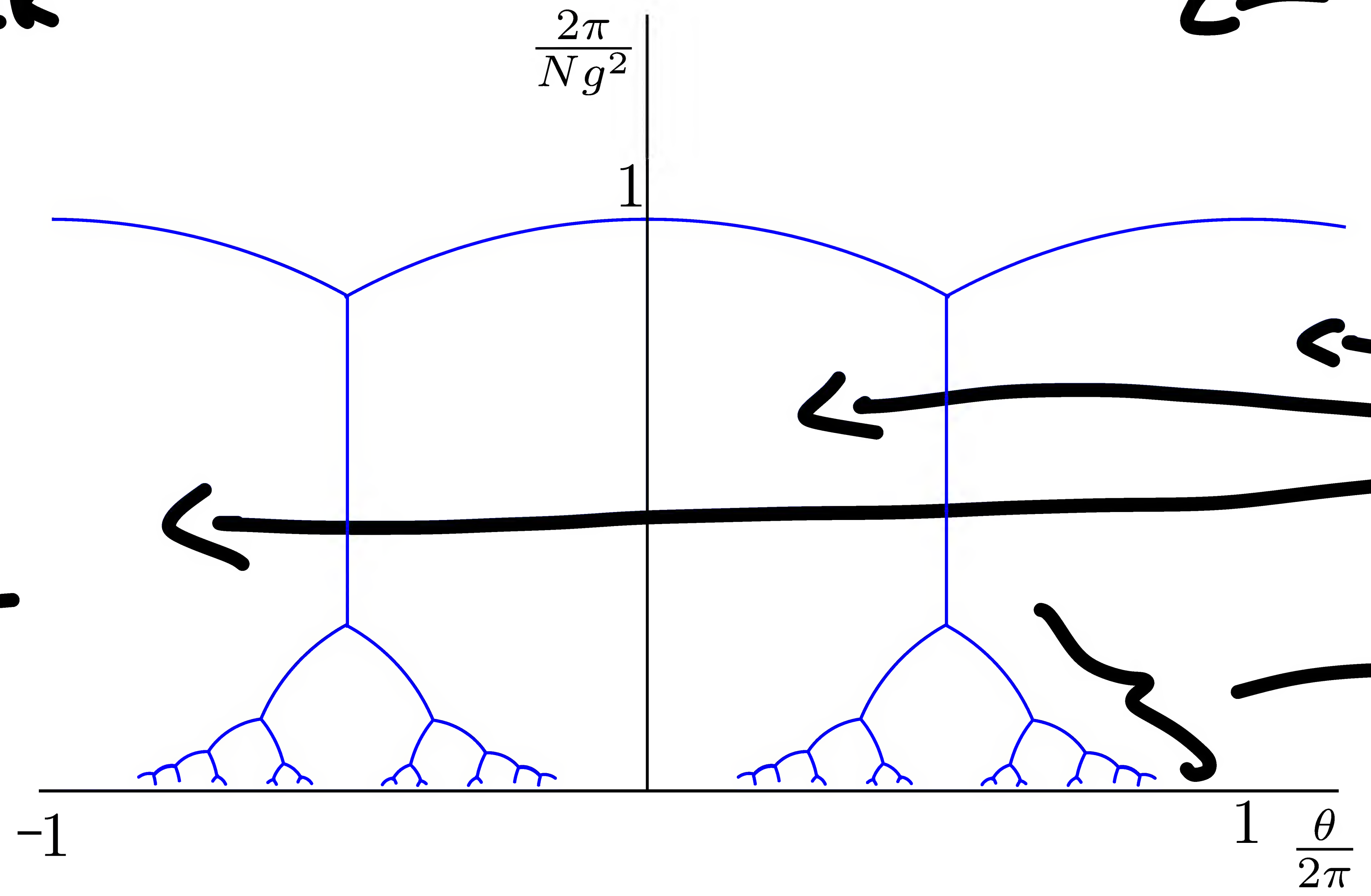
$$F_{(n,m)} \sim g^2 \left(N\left(n+\frac{\theta}{2\pi}m\right)\right)^2 + \left(\frac{2\pi}{g}\right)^2 m^2$$

$$= \frac{N}{\mathrm{Im}(\tau)} |n+m\tau|^2 \qquad \left(\tau = \frac{\theta}{2\pi} + i\,\frac{2\pi}{Ng^2}\right).$$

Predicted phases

weak ↑

↓ strong



← Higgs

← Confinement

Oblique confinement

Anomaly Topological aspects of CR model

't Hooft anomaly of CR model.

This model has the same anomaly with pure YM [GKKS '17]

B : Gauge field for $\mathbb{Z}_N$ 1-form symmetry

$$Z_{\theta+2\pi}[B] = e^{i\frac{N}{4\pi}\int B\wedge B} Z_\theta[B]$$

This anomaly can be used to constrain the possible phase diagrams.

$\theta \to \theta + 2\pi$: $\left(\theta\text{-term is } i\frac{N\theta}{8\pi^2}\int f \wedge f\right)$.

$$\Delta S = 2\pi i \ \frac{N}{8\pi^2}\int (f-B)\wedge(f-B)$$

$$= 2\pi i\left(\underbrace{\frac{N}{8\pi^2}\int f\wedge f}_{\in N\mathbb{Z}} - \underbrace{\frac{N}{4\pi^2}\int f\wedge B}_{\in \mathbb{Z}} + \frac{N}{8\pi^2}\int B\wedge B\right)$$

$$= i\,\frac{N}{4\pi}\int B\wedge B\,.$$

(Under this transformation, $H \to H\,W^{-N}$ by Witten eff.,
so $\{n_\mu, m_\mu\} \to \{n_\mu - m_\mu, m_\mu\}$.)

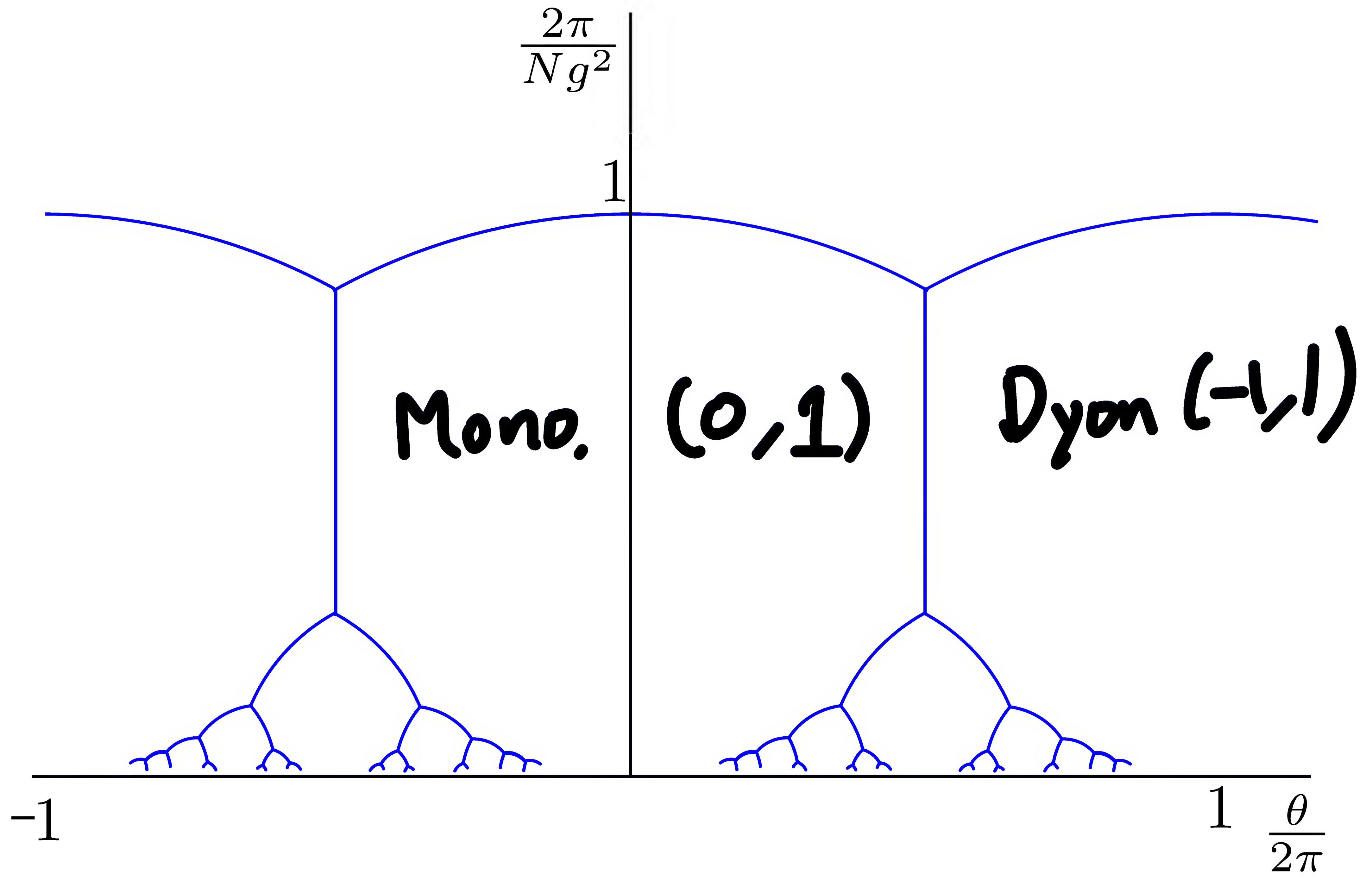
Anomaly matching in confined phase

$\theta \approx 0$: Monopole condensation

$$Z_0[B] \sim 1$$

$\theta \approx 2\pi$: Dyon condensation

$$Z_{2\pi}[B] \sim e^{i\frac{N}{4\pi}\int B\wedge B}$$



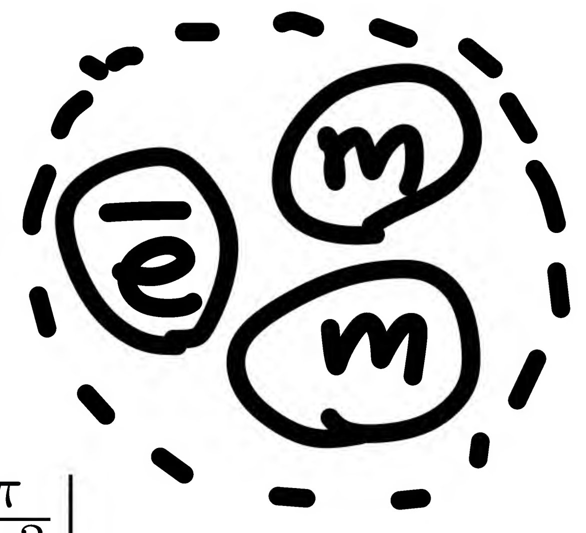
These two vacua are distinct as SPT phases with $\mathbb{Z}_N$ 1-form sym.

Exotic condensation : Oblique confinement.

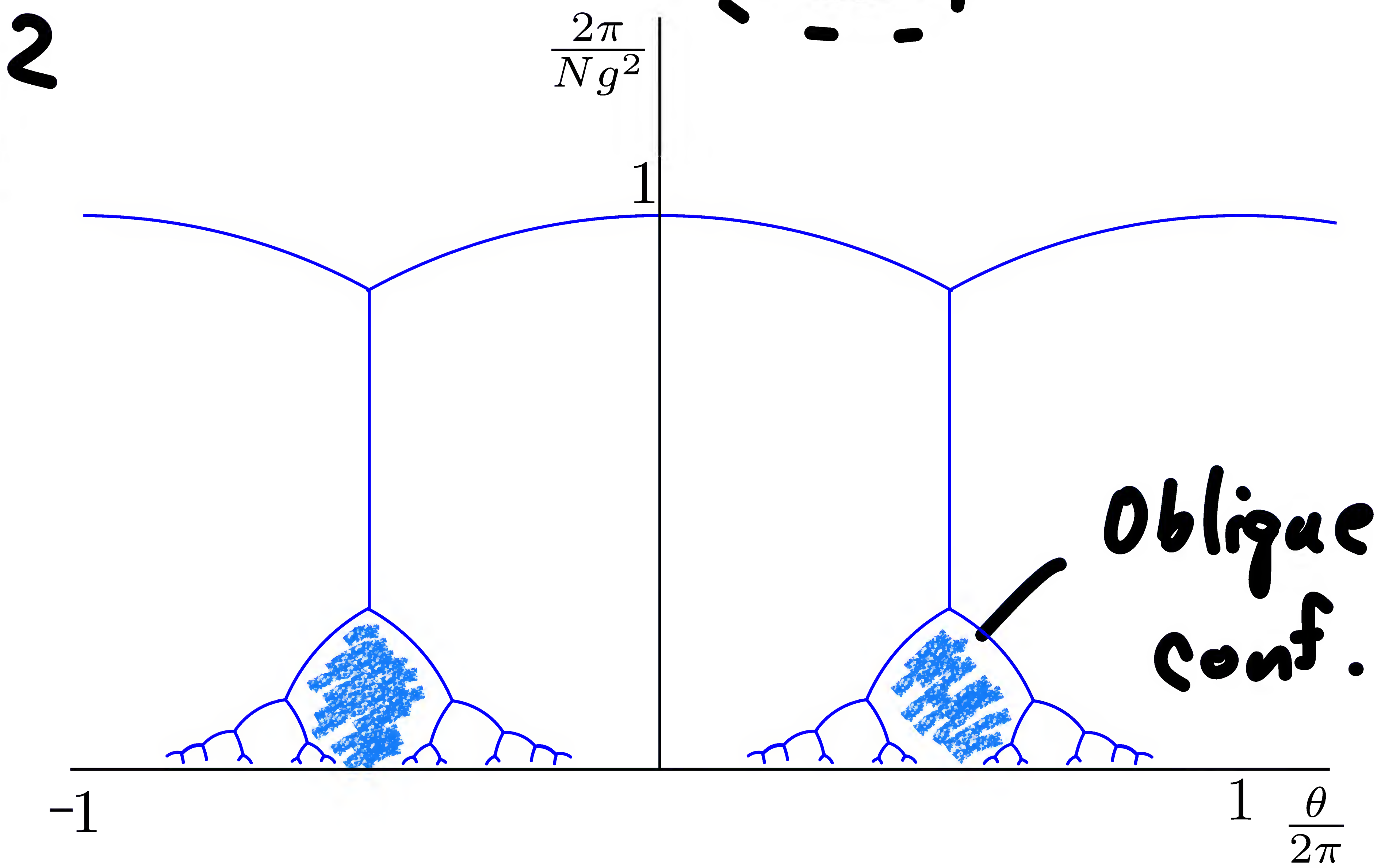
Around $\theta = \pi$, exotic condensation is more preferred :

$$F_{\text{mono}} = F_{(0,1)} \sim N^2 g^2 \Big(\underbrace{\frac{\theta}{2\pi}}_{=\frac{1}{2} \text{ at } \theta = 0} \Big)^2$$

Consider a composite particle $(-1, 2)$



$$F_{\text{oblique}} = F_{(-1,2)} \sim N^2 g^2 \Big(\underbrace{-1 + 2 \cdot \frac{\theta}{2\pi}}_{\substack{=0 \\ \text{at } \theta = \pi}} \Big)^2$$



Low-energy property of oblique confinement

<u>N: even int.</u> $\mathbb{Z}_N^{(1)} \xrightarrow{\text{SSB}} \mathbb{Z}_{N/2}^{(1)}$.

Only $W^{\frac{N}{2}}(C)$ is deconfined. $\to$ $\mathbb{Z}_2$ topological order.

<u>N : odd int.</u> All non-trivial Wilson loops are confined.

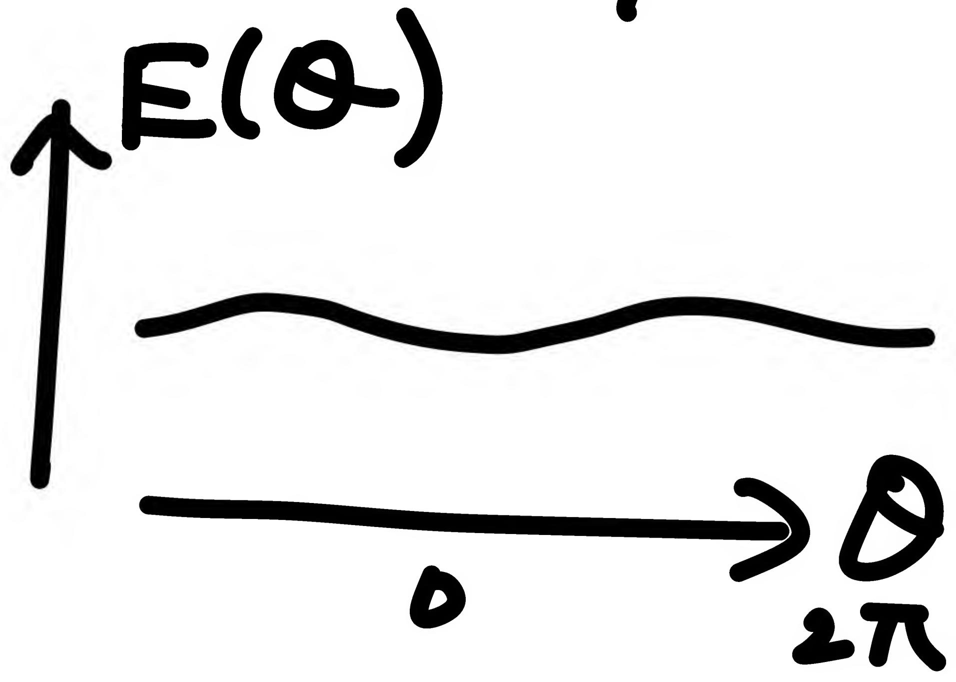
It's an SPT with $\mathbb{Z}_N^{(1)}$,

$$Z[B] \sim e^{i\frac{N-1}{2}\cdot\frac{N}{4\pi}\int B^2}.$$

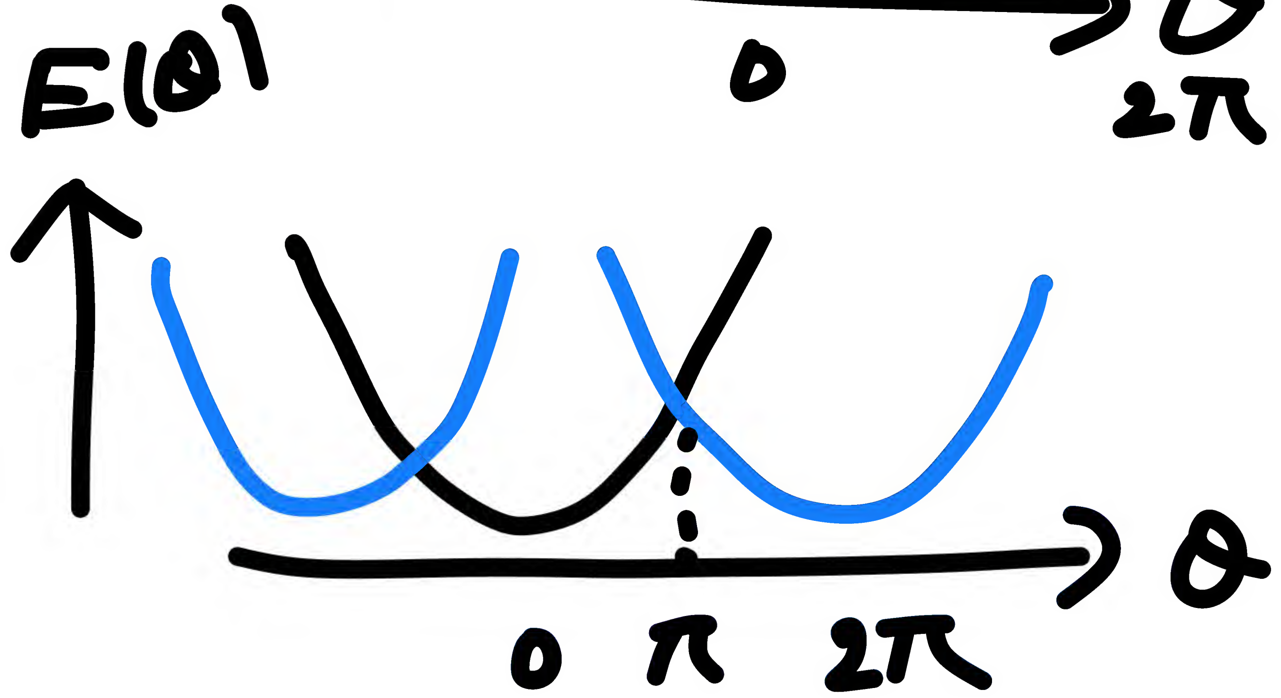
(cf. 't Hooft '81 for SU(2) gauge theory)

To match $\mathbb{Z}_N^{(1)} \times CP$ anomaly, we can have the following options, which are concretely realized in CR model:

- Higgs (or Coulomb) $\quad \mathbb{Z}_N^{(1)} \to 1$



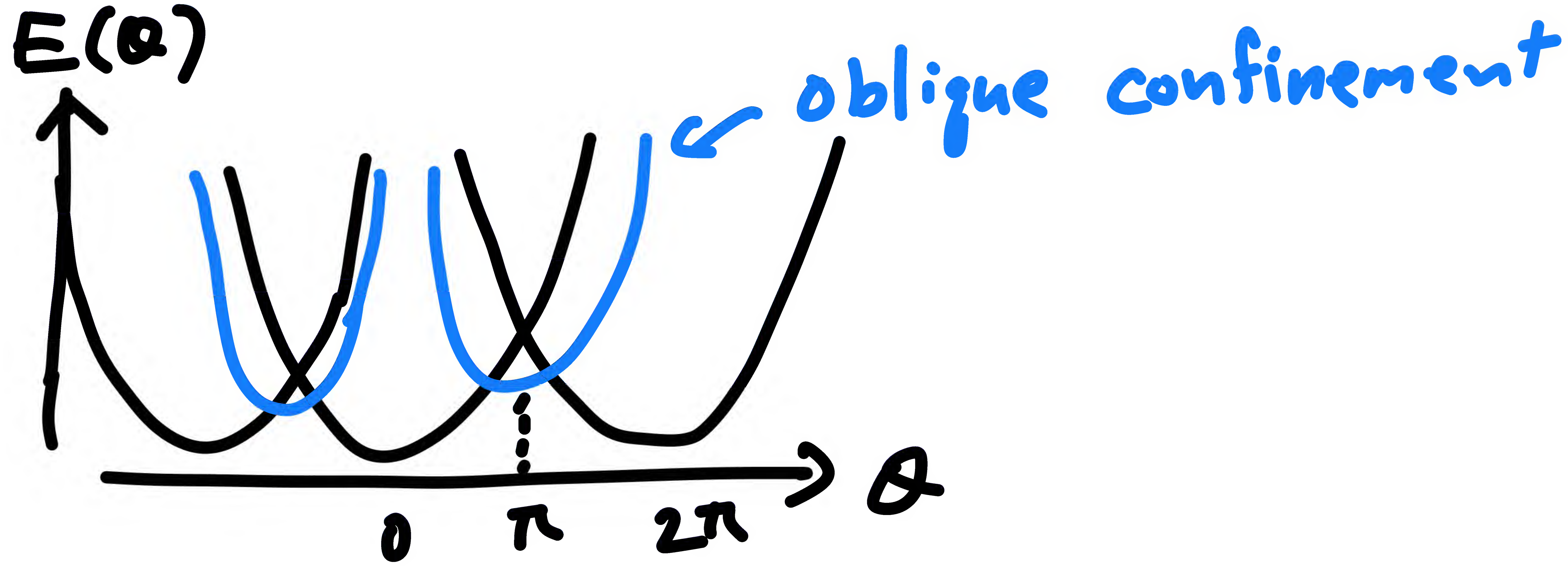
- Confinement $\quad \cancel{CP}$



- Oblique confinement

$\mathbb{Z}_N^{(1)} \to \mathbb{Z}_{N/2}^{(1)}$ (N : even)

$\mathbb{Z}_N^{(1)}, CPV$ (N : odd)



Self-duality $\quad SL(2,\mathbb{Z})$-duality and gravitational anomaly

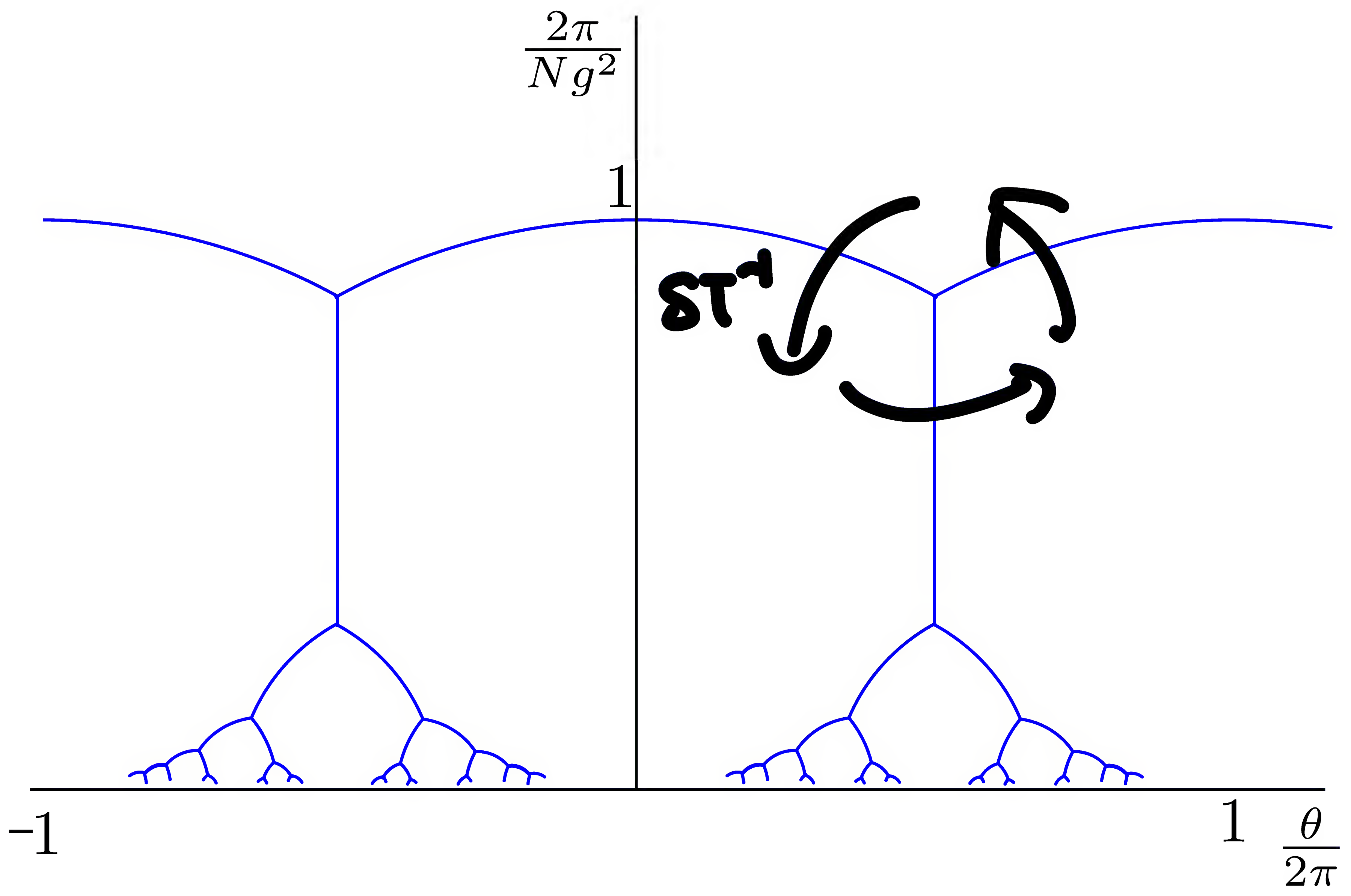
$SL(2,\mathbb{Z})$ elemag. duality of CR model

Free energy $F_{(n,m)} = \frac{1}{\mathrm{Im}(\tau)}|n+m\tau|^2$ is inv. under

$$SL(2,\mathbb{Z})\begin{cases} S: \ \tau \to -\frac{1}{\tau}, & (n,m)\mapsto(-m,n) \\ T: \ \tau\to\tau+1, & (n,m)\mapsto(n-m,m) \end{cases}$$

Especially, ST^{-1} has a fixed point

$$\tau = \tau_* = \frac{1}{2}+\frac{\sqrt{3}}{2}i.$$



This $SL(2,\mathbb{Z})$ is NOT a "self"-duality as 1-form sym. is not preserved. (Similar is true for $\mathcal{N}=4$ SYM cf. Gaiotto, Kapustin, Seiberg, Willett '14)

When $N=M^2$, we can obtain self-dual CR model by gauging $\mathbb{Z}_M^{(1)} \subset \mathbb{Z}_{M^2}^{(1)}$.

$$Z_{\text{gauged}} = \int \mathcal{D}a \exp\left(-\int\left(\frac{1}{2M^2g^2} da\wedge * da + i\frac{\theta}{8\pi^2} da\wedge da\right)\right)$$

$$\times \sum_{\{C_e, C_m\}} \tilde{W}^M(C_e)\ \tilde{H}^M(C_m).$$

⇝ Can we use this self-duality to constrain the phase str.?

Mixed $SL(2,\mathbb{Z})$ - gravity anomaly.

It's been known that Maxwell partition func. is a modular form, instead of being modular inv. (Witten '95, Verlinde '95) (Seiberg, Tachikawa, Yonekura '18).

$$\begin{cases} Z_{\text{Maxwell}}(\tau+1) = Z_{\text{Maxwell}}(\tau) \\ Z_{\text{Maxwell}}\left(-\frac{1}{\tau}\right) = \tau^{\frac{\chi-\sigma}{4}}\,\bar{\tau}^{\frac{\chi+\sigma}{4}}\, Z_{\text{Maxwell}}(\tau). \end{cases}$$

Here,

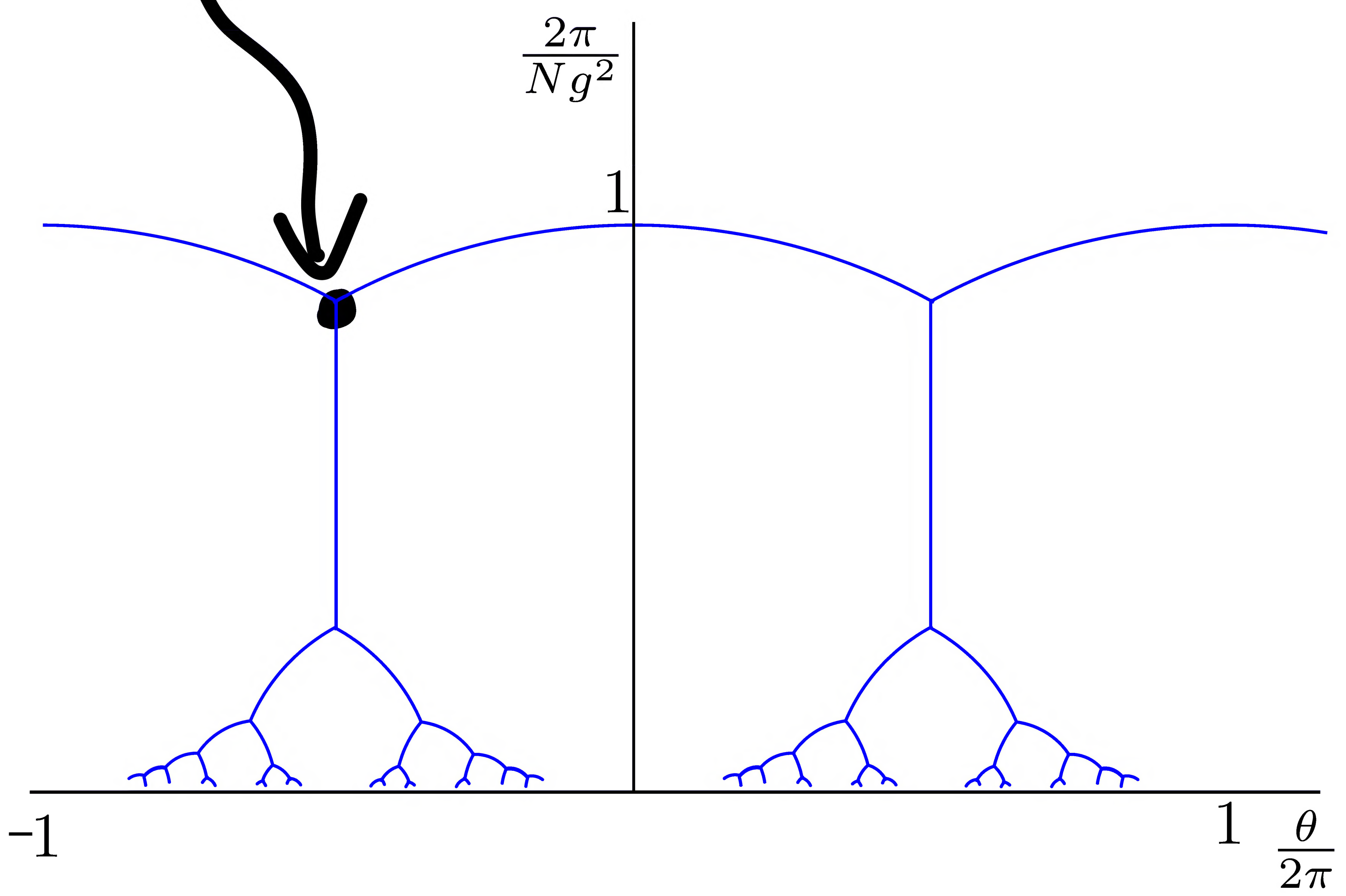
χ : Euler number

$\sigma = \frac{1}{3}\int \frac{1}{8\pi^2}\mathrm{tr}(R\wedge R)$: signature

We can then show that (on K3 surface)

$$(\mathbb{Z}_6)_{ST^{-1}} : Z \to \underbrace{e^{-\frac{2\pi i}{3}}}_{\hookrightarrow \text{ mixed gravitational anomaly.}} Z$$

$\Rightarrow$ Triple degeneracy by $\underline{(\mathbb{Z}_6)_{ST^{-1}} \xrightarrow{SSB} \mathbb{Z}_2}$ is required from anomaly.



Summary

- Confining vacua in YM have rich top. structures.
- Cardy-Rabinovici model provides a useful playground to understand those possibilities.
- We get better knowneldge on Oblique Conf.
- Self-dual CR model & its gravitational anomaly